

The Uniform Distribution:

A supplement to Triola Chapter 5, Section 5.2, and prelude to the Central Limit Theorem Excel exercise

Continuous random variables (see Triola, Section 5.2) that have equally likely outcomes over their range of possible values have a **uniform probability distribution**. Suppose the random variable  $x$  can assume values only in the interval  $a \leq x \leq b$ . Since the total probability over all values of  $x$  must be equal to 1, the probability distribution,  $p(x)$ , can be described by the definition

$$p(x) = 1/(b-a) \text{ for } a \leq x \leq b$$

and  $p(x) = 0$  for  $x < a$  and  $x > b$

as shown in Figure 1. Note that the total probability is given by the area of the shaded rectangle:

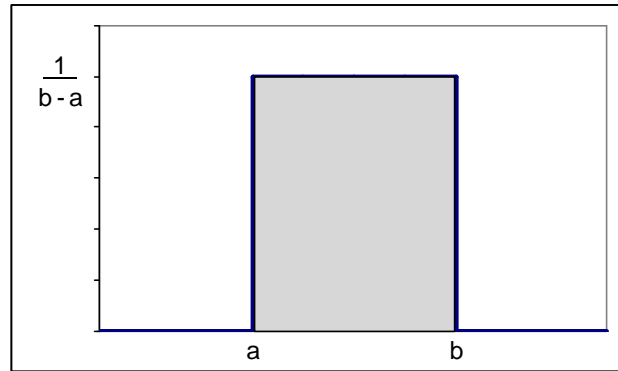


Figure 1

$$\text{Area} = (b-a) \times 1/(b-a) = 1$$

and the probability of finding a value in the range  $c$  to  $d$ , where  $c$  and  $d$  lie between  $a$  and  $b$  is

$$(d-c)/(b-a)$$

The uniform distribution has the following properties:

$$\text{mean} = m = \frac{a+b}{2}; \text{ standard deviation} = s = \frac{b-a}{\sqrt{12}}$$

A random number generator will generate numbers uniformly between 0 and 1; thus a random number generator can be considered to be equivalent to a uniform distribution with  $a = 0$  and  $b = 1$ .

ASSIGNMENT:

Calculate the mean and standard deviation of the numbers produced by a random number generator:

A. Mean =  $\mu$  = \_\_\_\_\_

B. Standard deviation =  $s$  = \_\_\_\_\_

If a random number generator is used to produce 16,000 numbers, how many would you expect to occur in the following ranges?

C. 0 to 0.05: \_\_\_\_\_

D.  $(\mu - 0.05)$  to  $\mu$ : \_\_\_\_\_

E.  $\mu$  to  $(\mu + 0.05)$ : \_\_\_\_\_

F. 0.95 to 1.00: \_\_\_\_\_

G. Bonus: If many samples of 16 or 1000 random numbers are taken, what would be the mean ( $\mu_{\bar{x}}$ ) and standard deviation ( $s_{\bar{x}}$ ) of the sample means?

	n=16	n=1000
$\mu_{\bar{x}} =$		
$s_{\bar{x}} =$		

**Turn in this sheet, with the calculations completed, along with the results of your completed Central Limit Theorem Excel exercise.**